

# Computational Hemodynamics

During the past two decades, hemodynamics has played an important role in the understanding of arterial diseases and in the regulation of cellular biology in both normal and diseased arteries. Vascular disease, including atherosclerosis, aneurysms, and plaque disruption, is one of the leading cause of deaths in the United States. Computational fluid dynamics is emerging as a more powerful method to understand this natural phenomenon compared to in vivo or in vitro experiments.

## Transitional Blood Flow Simulation with Spectral Finite Elements

While the natural flow state in the vessels is laminar, it is possible to have a transition to a weakly turbulent state in the presence of stenoses or after a surgical procedure such as an arteriovenous graft implementation. The transition to turbulence induces a sudden change in the range of spatial and temporal scales in the solution, resulting in a need for two to three orders of magnitude increase in computational resources for the same physical-time simulation. The flow physics in the turbulent case is dominated by the convection of momentum with relatively little diffusion. A typical Reynolds number for healthy arteries is  $\sim 350$  but may be as high as 1000-3000 in the transitional case. For simulation in the high Reynolds number regimes where physical dissipation is small, high-order numerical discretization that has minimal numerical dispersion and dissipation per grid point is essential.

Our primary effort has focused on the development of the spectral element method (SEM), which offers geometric flexibility, rapid numerical convergence, and tensor product efficiency. With the SEM the domain is decomposed into  $E$  curvilinear hexahedral elements, and the solution within each element is represented as an  $N$ th-order tensor-product nodal-based polynomial. For 3D, there are approximately  $EN^3$  gridpoints in the entire domain. For similar resolution, SEM requires a two to three orders of magnitude smaller number of elements than does the finite element methods.

## All Hex-Meshing Algorithm

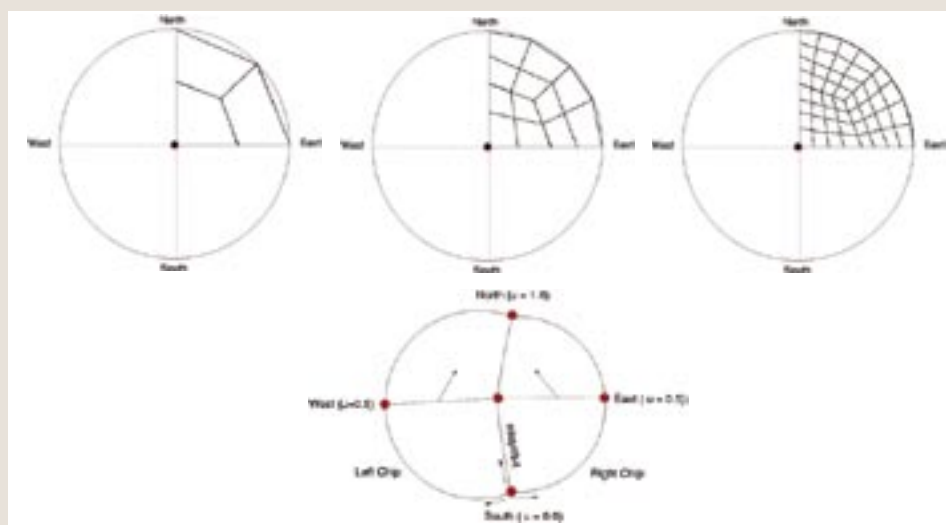
We have developed an automated all-hex strategy for bifurcation geometries characterizing arteries. The key components of our approach are the use of a natural coordinate system, derived from solutions to Laplace's equations, that follows the tubular vessels, and use of a tripartitioned-based mesh topology that leads to very high quality meshes in each of the branches. The method is designed for situations where the required number of hexahedral elements is relatively small ( $\sim 1000-4000$ ), as required in spectral element-based simulation. This reduction poses challenges for automated hex-mesh generation because (1) there are relatively few interior elements over which mesh optimization can be used to absorb topological corrections, and (2) there are sharp curvatures and rapid variation in diameters.

## Performance Results

- $E = 2640$ ,  $N = 10$  ( 2.6 million velocity points)
- 9500 time steps/cardiac-cycle
- 22 hours on 1024 nodes of the Blue Gene/L



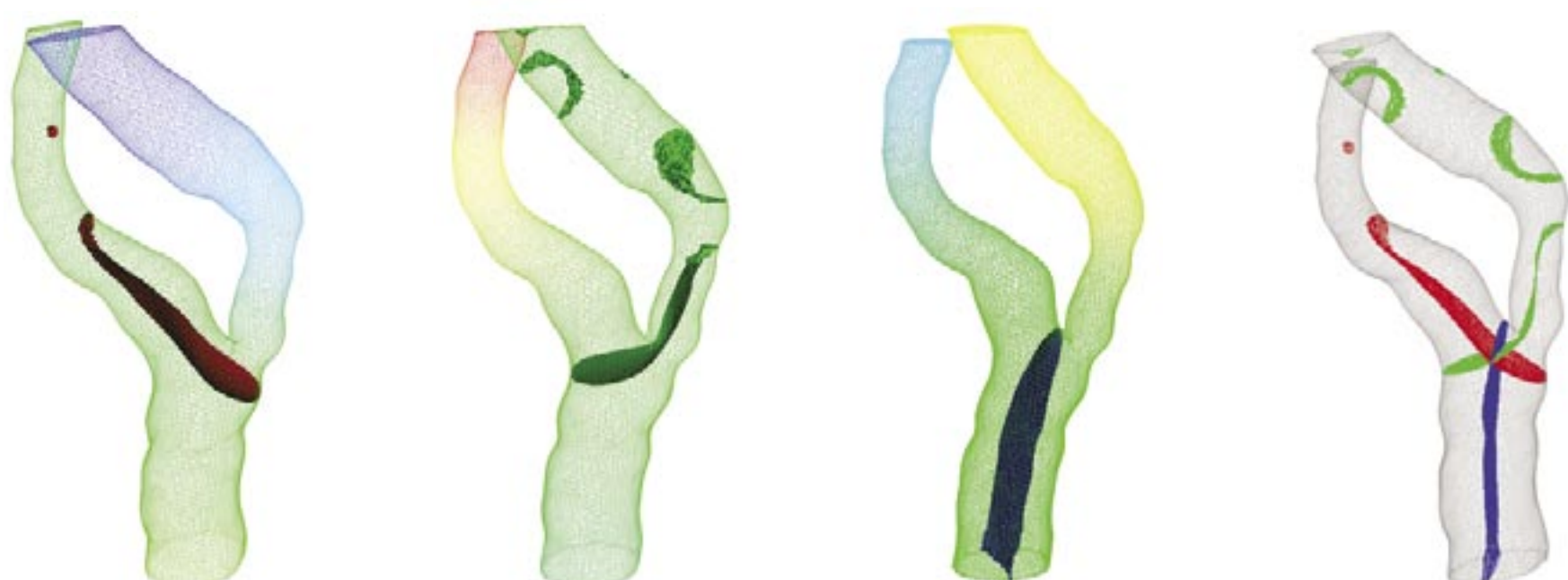
A zoomed picture of principal chips (left and center) and a half-cylinder.



Example of mesh templates, with the topology of each chip shown in the bottom figure.



Left: Chips in artery Right: The final very high quality hex mesh.



Isosurfaces from the three heat conduction problems.